Numerical Modeling of Left-Handed Metamaterials

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U.S. Department of Energy

Lawrence
Livermore
National
Laboratory

November 6, 2001

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Abstract The EIGER method of moments program with periodic Green's function was used to model a periodic array of strips and split-ring resonators. Left-handed propagation due to negative index of refraction is demonstrated in a frequency band. The effective material parameters versus frequency are extracted from the EIGER solution.

Introduction

There has been considerable interest recently in metamaterials that exhibit negative index of refraction. Such "left-handed" materials, having negative permittivity ϵ and negative permeability μ, were considered theoretically by Veselago [1]. If only one of μ or ϵ is negative the refractive index n is imaginary, and waves cannot propagate. When both are negative n is real, and energy considerations lead to the conclusion that n must be negative so that the phase velocity of a wave is in the opposite direction to power flow. A wave incident on a slab of such a material will be diffracted across the normal, rather than only toward the normal as in a normal medium. Also, the Doppler shift is expected to be reversed, and rather than radiation pressure there will be radiation tension. A number of applications have been proposed for such materials, including design of band-pass filters and beam steerers. A flat slab of the material could focus waves, forming a "perfect lens" that would focus both propagating and evanescent waves and could permit sub-wavelength focusing of a point source.

Materials with negative μ and ϵ do not exist in nature, but composite structures have been designed with metallic inclusions to produce the desired properties. A periodic array of parallel wires is known to exhibit negative ϵ below some "plasma frequency" and positive values for higher frequencies in a Debye response curve [2, 3]. Negative permeability has been demonstrated in an array of cells containing split-ring resonators (SRRs) [4]. The permeability fits a Lorentz model, with negative values in the resonance region of the split rings. Wires or narrow strips and SRRs have been combined to obtain simultaneously negative μ and ϵ in a limited band. Such materials have been constructed by Smith and others [5, 6] and shown to produce left-handed propagation.

Some attempts have been made to model left-handed materials combining strips and SRRs, particularly using the Finite Difference method and averaging fields [7]. In this paper we present results from modeling periodic strips and SRRs in the frequency domain with the EIGER program with periodic Green's function [8].

The EIGER model and data analysis method

The square form of the SRRs used by Shelby et al. [9] was chosen for modeling with EIGER, as shown in Figure 1. The dimensions of the SRRs were modified to bring the resonance down to about 11 GHz, below the 13.3 GHz plasma frequency of the strip array. Using Shelby's dimensions without the fiberglass backing on his model resulted in the SRRs resonating at 16.5 GHz so that negative μ and ϵ regions would not have overlapped.

A slab of the metamaterial 12 cells deep, extending from $z = 0$ to $z = 0.06$ m, was modeled by generating data for 12 elements with spacing of 5 mm in the z direction and making this cell periodic in the x and y directions with a lattice constant of 5 mm. The strip of 12 SRRs and

conducting strips is shown in Figure 2. This cell is repeated in the periodic array so that the strips become continuous. The SRRs and strips were oriented in the x-z plane, so that a wave incident from the $+z$ direction hit them edge on. The incident wave had $\theta = 0.01°$ since some non-zero x or y component of the wave vector is needed in the EIGER solution for a periodic medium.

In order to model a periodic structure EIGER must be run for a "layered periodic" medium, a legacy of the solution set up for periodic microstrip structures. Phantom layers with free space parameters were modeled to satisfy this requirement. All materials and field evaluation points must be contained in the bounded layers, and it was found that the code can produce numerical overflows if the evaluation points are too far from a layer interface. To avoid overflow, layer interfaces were placed between the elements at $z = -0.03, 0., 0.015, 0.03, 0.045, 0.06$ and 0.08 m with free-space parameters for all layers. This allowed computing the near field through the slab from $z = -0.03$ m to 0.08 m.

With an EIGER model the field can be computed along a path through the slab, including reflected and transmitted fields and fields within the slab. It is possible to solve for the values of μ and ϵ of the slab material by using only values of the reflected and transmitted field, under the assumption that the slab supports only waves of the form e^{-jk_2z} and e^{+jk_2z} where $k_2 = (\omega/c)\sqrt{\mu_2\epsilon_2}$. The slab will be considered medium 2 and the outside medium 1. A convenient form of the solution for μ_2 and ϵ_2 was given by Nicolson, Ross and Weir (NRW) [10]. However, we wanted to characterize the material including stop-bands where n is imaginary, and the NRW solution becomes indeterminate when the transmitted field goes to zero. Also, this result may be difficult to use, since it contains several square roots for which the branch cuts must be chosen properly, and one must know or guess the number of wavelengths within the slab and then verify the guess.

As an alternate method of determining μ_2 and ϵ_2 we used the field within the slab and applied the Generalized Pencil of Functions method (GPOF) [11] to resolve the field into exponential waves of the form $E(z) = \sum_{i=1}^{N} \alpha_i e^{\gamma_i z}$. N was set to 10 in the GPOF solution, but only the first one or two waves were significant. The wave with the largest α_i is chosen, and the wavenumber in the slab is then $k_2 = j\gamma_i$ where $j = \sqrt{-1}$. From this result one gets $\sqrt{\mu_2\epsilon_2} = k_2/k_1$. A value for $\sqrt{\mu_2/\epsilon_2}$ can be obtained from one value of the reflected field in front of the slab. From the reflected field of an ideal slab with width D the result is

$$\frac{\eta_2}{\eta_1} = \sqrt{\frac{\mu_2}{\epsilon_2}} = \frac{E_{r0}(1 + e^{-j2k_2D}) + \sqrt{1 + e^{-j2k_2D}(e^{-j2k_2D} + 4E_{r0}^2 - 2)}}{(E_{r0} - 1)(e^{-j2k_2D} - 1)} \tag{1}$$

where E_{r0} is the reflected field at the front surface of the slab, which is the reference point for zero phase of the incident wave. Trials showed that good results could be obtained when the effective surfaces of the slab were taken at the outer surfaces of the first and last cell when the elements are centered in the cells. The total field was evaluated at some distance z_1 in front of the slab to avoid surface effects. The reflected field with phase corrected to the surface of the slab at z_0 is then evaluated as

$$E_{r0} = (E_{t1} - e^{-jk_1z_1})e^{jk_1(z_1-2z_0)}$$

where the incident field is one volt per meter with zero phase at $z = 0$.

A possible problem in this method of getting the medium parameters is that the field computed through the slab may show some effects of the local structure of the medium. The field was computed along a path half way between the SRR/strip structures to minimize such field perturbations.

Modeling results

Some results for magnitude and phase of the electric field through the slab of SRRs and strips are shown in Figures 3, 4 and 5. The wave is attenuated in the slab until a frequency around 10.9 GHz due to negative ϵ_2 and positive μ_2. At 10.9 GHz μ_2 becomes negative and the wave propagates. The phase plot is distorted by forward and reflected waves, but the predominate slope indicates the negative phase velocity (to the right) for the largest wave. Since the largest wave must be carrying power to the left, in the direction of the incident wave, there appears to be "left-handed propagation" taking place. This behavior continued to just below 11.4 GHz. Fields in the slab from 11.4 to 12 GHz are shown in Figures 4 and 5. In this range μ_2 is small but positive again while ϵ_2 remains negative. The wave shows a small attenuation with very high phase velocity. Ziolkowski [12] has shown that materials with similar behavior over a broad band can be used to transfer information at faster than the velocity of light. He points out that this is a near field effect and does not violate causality, but could be used to transmit data over short distances. Above about 12 GHz, ϵ_2 becomes positive and normal propagation resumes in the strip and SRR metamaterial.

The reflected and transmitted fields for the slab of strips and SRRs are plotted in Figure 6 and the wavenumber k_2 obtained from the GPOF analysis of field in the slab is shown in Figure 7. The values of μ_2 and ϵ_2 obtained from these results are plotted in Figure 8. The μ_2 and ϵ_2 values were fit to Lorentz and Debye response functions, respectively, as

$$\tilde{\mu}_2(f) = 1 - \frac{a}{1 + b/f + c/f^2} \qquad \text{and} \qquad \tilde{\epsilon}_2(f) = 1 - \frac{d}{f^2 + ef}.$$

The constants for μ_2 were $a = -0.0103 - j0.000049$, $b = 19.6 - j0.0088$ and $c = 95.1 - j0.0925$ and for ϵ_2 they were $a = 56.3 - j2.72$ and $b = -0.478 + j0.0195$

While the structure of Figure 2 is seen to produce left-handed propagation in a limited frequency band, there is considerable reflection of the incident wave. The reflection could be reduced or eliminated if μ_2 and ϵ_2 were closer to –1 in the frequency band. One application proposed for left-handed media is to form a "perfect lens". Ziolkowski and Heyman [13] have pointed out that to obtain a perfect focus it is required that $\mu_2 = \epsilon_2 = -1$ without dispersion. Dispersion seems unavoidable with this structure, but the condition on μ_2 and ϵ_2 might be approached by scaling the structure. Extrapolation of the results for ϵ_2 in Figure 8 with the Debye curve fit predicts that ϵ_2 would reach -1 at about 8.7 GHz, while $\mu_2 = -1$ at around 11.2 GHz. Hence the SRRs were scaled up from $w = 2.9$ mm to 3.7 mm to attempt to bring the frequency for $\mu_2 = -1$ down to 8.7 GHz. The scaled structure is shown in Figure 9, and results are plotted in Figures 10 through 12. With this simple attempt at scaling, μ_2 and ϵ_2 do not hit -1 quite simultaneously, but they are close enough to greatly reduce the reflected field from about 8 to 8.25 GHz.

Conclusions

Results from the EIGER solution utilizing the periodic Green's function demonstrate left-handed propagation in the medium of strips and SRRs. Analysis of the near fields appears to provide accurate results for the effective permittivity and permeability of the metamaterial. Modeling permits easy modification of the structure parameters to allow tuning and optimization of the material response and could permit design of more effective structures.

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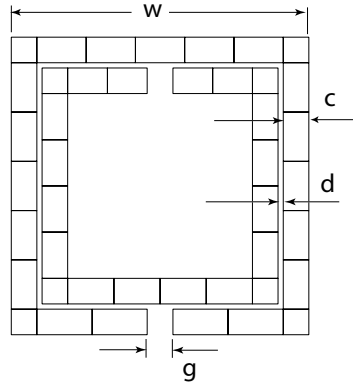
Fig. 1 Dual split-ring resonator (SRR) with $c = 0.25$ mm, $d = 0.05$ mm, $g = 0.25$ mm and $w = 2.9$ mm. The division into rectangular patches for the EIGER code is shown.
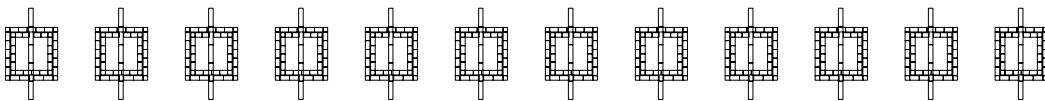

Fig. 2 SRRs and strips combined in a row of 12 elements that is made periodic in the directions vertical and normal to the page so that the strips become continuous in the vertical direction. The strips are offset by 0.02 mm in front of the SRRs. The wave is incident from the right with $\vec{E}$ vertical.
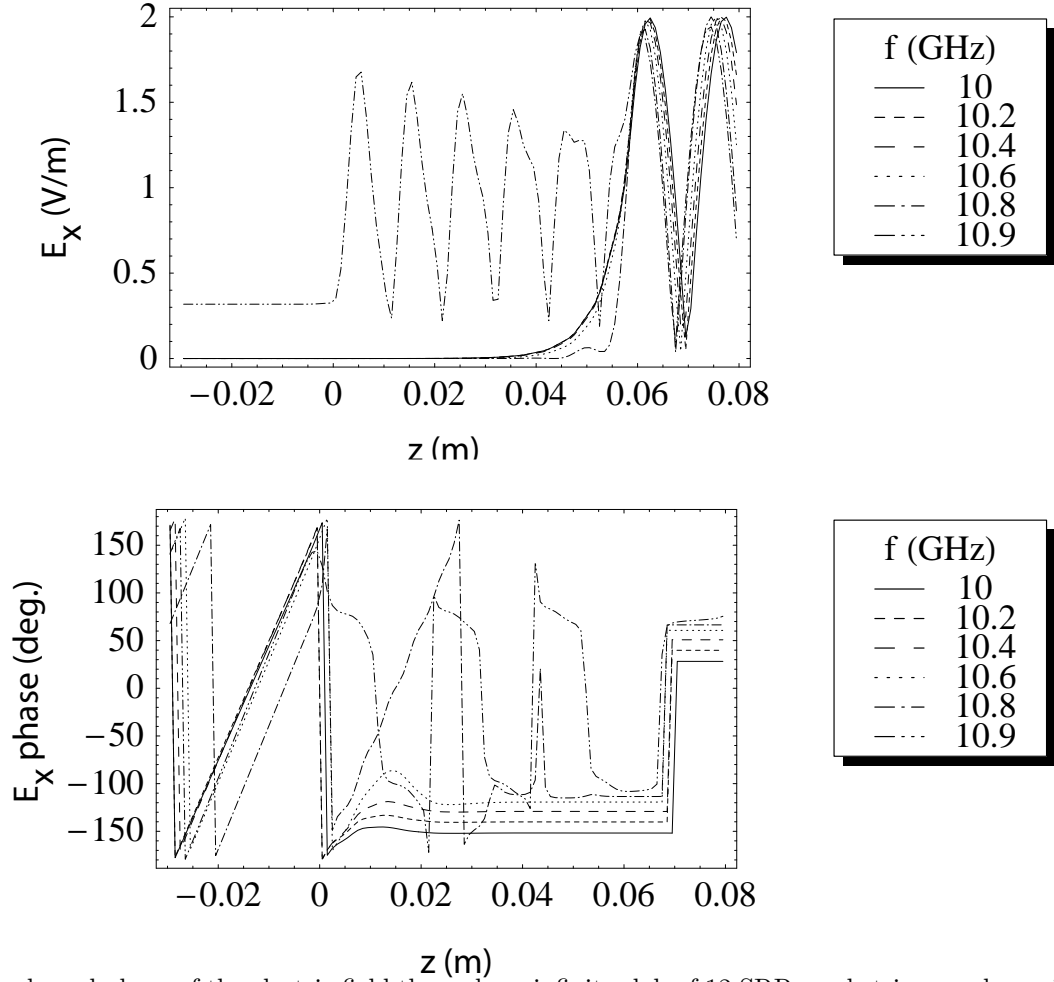
Fig. 3 Magnitude and phase of the electric field through an infinite slab of 12 SRRs and strips, as shown in Figure 2. The wave is incident from the right and the slab extends from $z = 0$ to 0.06 mm.
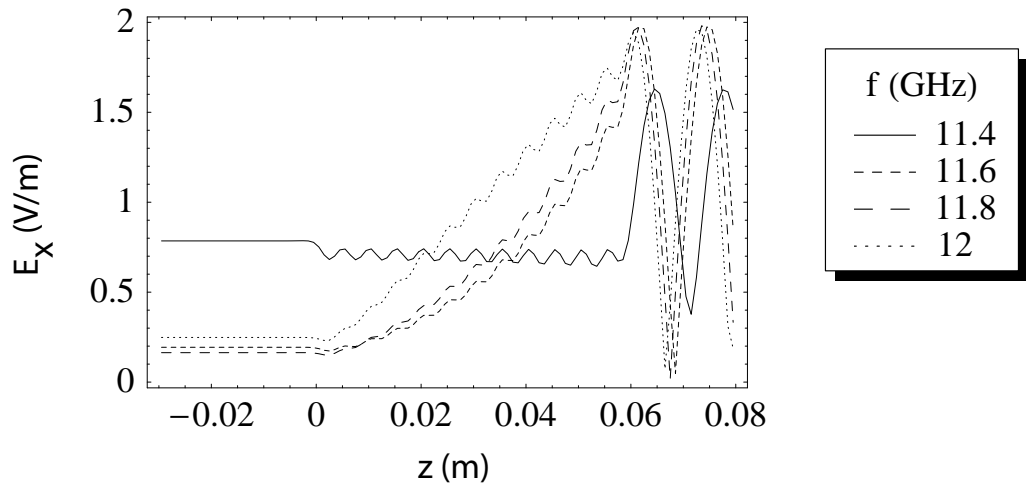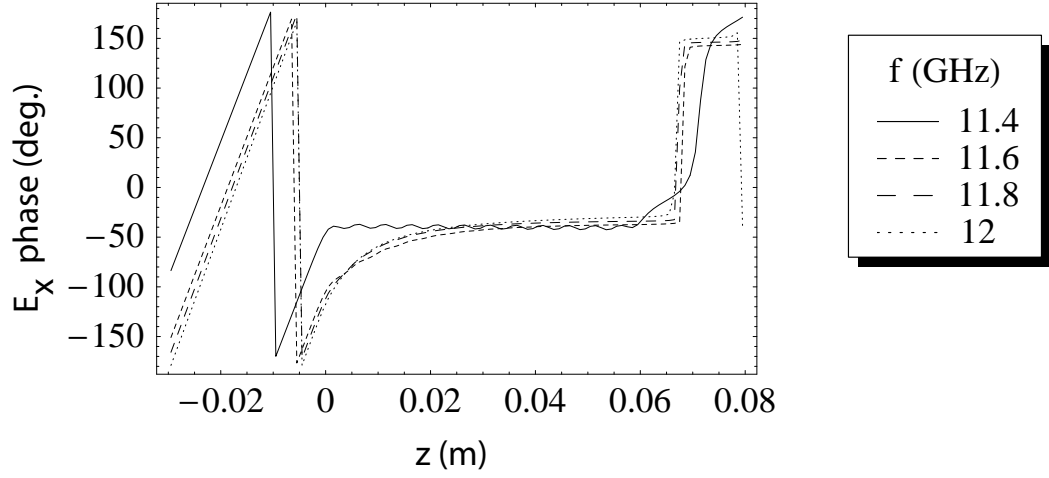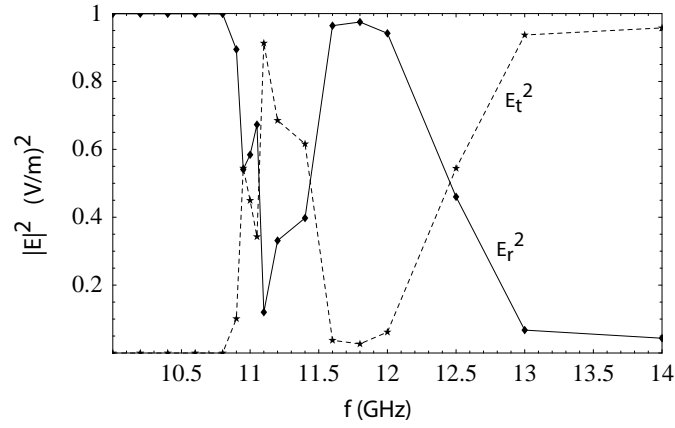

Fig. 4 Magnitude of the electric field through an infinite slab of 12 SRRs and strips, as shown in Figure 2. The wave is incident from the right and the slab extends from $z = 0$ to 0.06 mm.

Fig. 5 Phase of the electric field through an infinite slab of 12 SRRs and strips, as shown in Figure 2. The wave is incident from the right and the slab extends from $z = 0$ to 0.06 mm.

Fig. 6 Magnitude squared of the reflected field (E_2^2) and transmitted field (E_t^2) for a wave incident normal to a slab of 12 SRRS and strips.
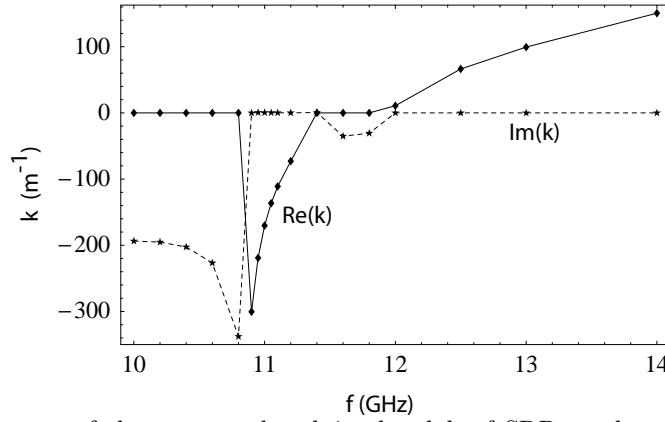

Fig. 7 Real and imaginary parts of the wavenumber k in the slab of SRRs and strips, as determined from the EIGER solution for fields.
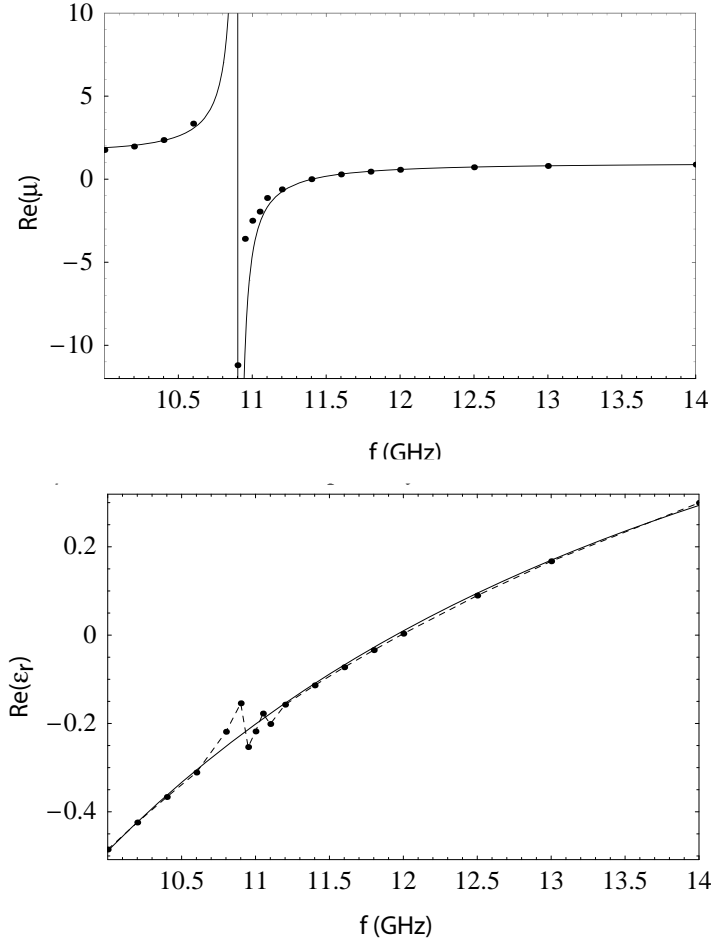
Fig. 8 Frequency dependence of μ_2 and ϵ_2 for metamaterial of strips and SRRs from Figure 2. The points are values from processing the EIGER results and solid lines represent Lorentz and Debye response functions fit to the EIGER results for μ_2 and ϵ_2, respectively.
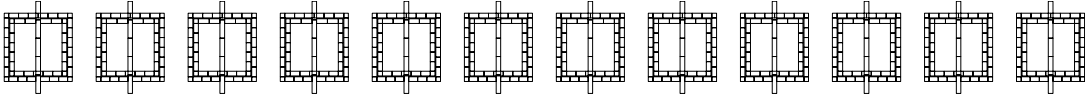

Fig. 9 Twelve SRRs and strips with the SRRs scaled to resonate around 8 GHz. Dimensions of the SRRs are the same as in Figure 1, except that w has been increased to 3.7 mm.
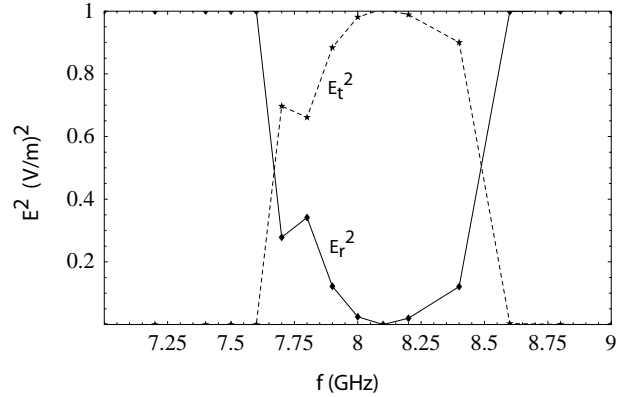

Fig. 10 Magnitude squared of the reflected field (E_2^2) and transmitted field (E_t^2) for a wave incident normal to a slab of 12 SRRS and strips formed from the elements of Figure 9.
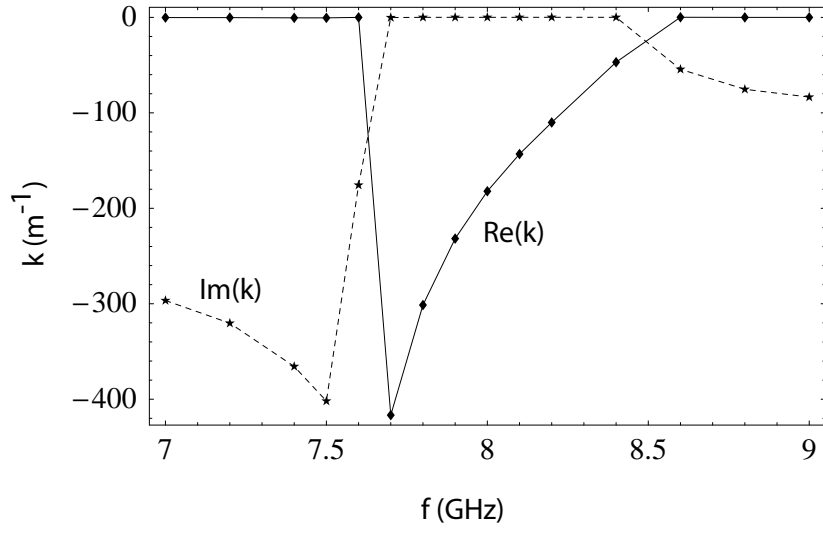
Fig. 11 Real and imaginary parts of the wavenumber k in a slab of SRRs and strips, as determined from the EIGER solution for fields. The slab is formed from the elements of Figure 9 with SRRs scaled to resonate around 8 GHz.
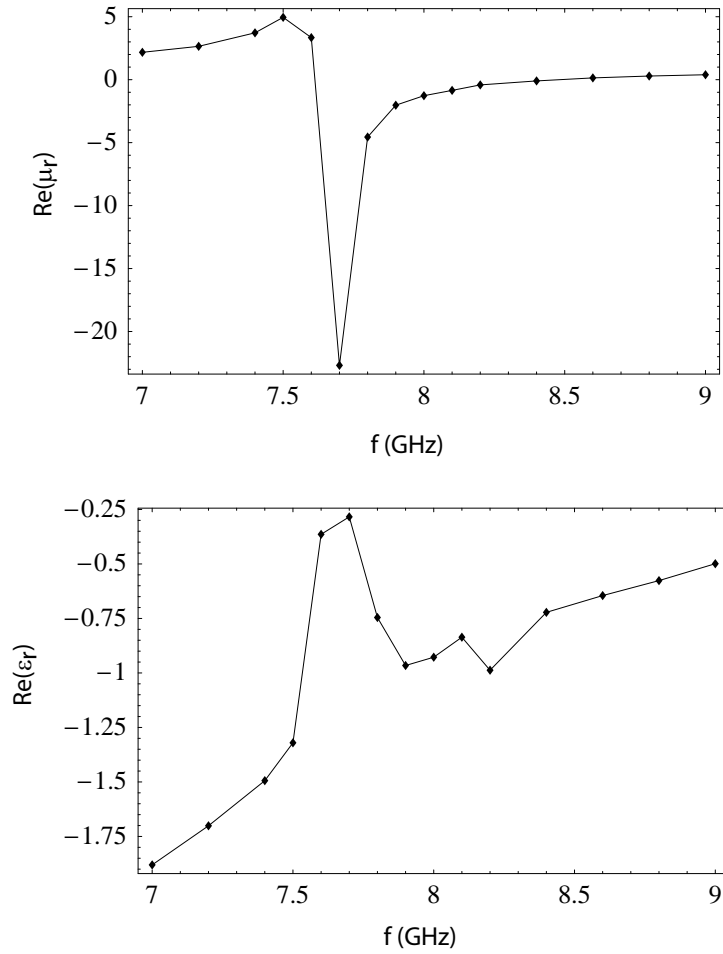

Fig. 12 Relative permeability μ_2 and permittivity ϵ_2 in a slab of SRRs and strips formed from the elements shown in Figure 9.